Daniel BUC, Tomáš KLIEŠTIK

RANDOM WALK - ITS DESCRIPTION & USAGE

Summary
This article deals with one of the most popular capital market approach – Random walk theory. There are fundamental elements, features and one basic example of this attitude described in here. Random Walk Theory denies other analysis, such as psychological, technical or fundamental, because of more reasons. These could be for example, bad or useless information, buy and sell timing and others. A protagonist of this theory says that it is not possible to outperform a particular market if any additional risk is assumed. On the other hand, critics of Random walk theory contend that assets do maintain price trends – there is a chance to outperform the market if the selecting exit and entry points for investments are carefully selected.

INTRODUCTION
The random walk theory is based on the fact that market and securities prices are random and not influenced by past events. This theory also states that all methods of predicting stock prices are futile in the long run. Professor Burton G. Malkiel calls the notion of intrinsic value\(^1\) undependable because it relies on subjective estimates of future earnings using factors like expected growth rates, interest rates, dividend payouts, estimated risk, and others.

1. RANDOM WALK FEATURES
In modern econometric and financial theory, stochastic differential equations are successfully used to describe the financial market and macroeconomic indicators development. Stochastic (linear trend) component of the differential equation is represented by Brownian motion or otherwise mathematically Wiener process belonging to Lévy processes which various modifications due to their features excellently approximate volatility and trend of asset prices. Random path model is named as Random Walk or a geometric Brownian motion.

1.1. Random walk vs. prediction methods
The random walk theory also considers technical analysis undependable because, according to Malkiel, chartists buy only after price trends are established and sell only after price trends are broken; essentially, the chartists buy or sell too late and miss the boat. According to the theory, this happens because stock prices already reflect the information by the time the analyst moves on the stock. Experts in this field also note that the widespread use of technical analysis reduces the advantages of the approach.

\(^{1}\) Intrinsic value is calculated by summing the future income generated by the asset and discounting it to the present value.
Further, Malkiel finds fundamental analysis flawed because analysts often collect bad or useless information and then poorly or incorrectly interpret that information when predicting stock values. Factors outside of a company or its industry may affect a stock price, rendering further the fundamental analysis irrelevant.

1.2. Forms of random walk

There are two forms of the random walk theory. In both forms, the rapid incorporation of information is disadvantageous for investors and analysts.

1. The semi-strong form states that public information will not help an investor or analyst select undervalued securities because the market has already incorporated the information into the stock price.

2. The strong form states that no information, public or private, will benefit an investor or analyst because even inside information is reflected in the current stock price.

2. MATHEMATICAL EXPLANATION OF RANDOM WALK

It is possible to model for example a random prices and exchange rates development through Random Walk model. This way of modeling has no tendency to return to its mean value.

If the considered asset $P$ with initial price $P_t$ where the random component is taken into account, we express the dynamics of asset prices at the time, which is expressed by the stochastic differential equation

$$dP_t = \mu * P_t * dt + \sigma * P_t * dz,$$

that represents a geometric Brownian motion defined as Ito process for $t \geq 0$, $\mu$ is the growth rate (average yield), $\sigma$ is the volatility. This process can be written in a way which indicates that the yields of particular asset are modeled by using the deterministic (drift) and a random (diffusion) component in the form

$$dP_t = \mu * dt + \sigma * dz$$

In some methodologies that are focused on risk quantification and diversification the logarithmic prices $p_t = \ln P_t$ are modeled that have normal distribution in the form $N(0; \sigma^2)$. Geometric Brownian motion with logarithmic prices can be then described as

$$dp_t = d\ln P_t = \tau * dt + \sigma * dz,$$

where the logarithmic price has the deterministic coefficient $\tau$ in form $\tau = \mu - \frac{\sigma^2}{2}$ and random coefficient $\sigma$.

Deterministic coefficient $\tau$ represents the average rate of profit resulting from price growth of relevant asset. It is estimated as a regressive model through statistical estimation – least squares method,

$$\sum_{t=1}^{T} \varepsilon_t^2 \to min,$$

where $\varepsilon_t$ is residual deviation which indicates the difference between the actual and estimated yield

$$\varepsilon_t^2 = \sum_{t=1}^{T}(r_t - \tau * P_t)^2$$
The random coefficient $\sigma$ determining the standard deviation is calculated as a square root from residual deviation,

$$
\sigma = \sqrt{\frac{1}{N} \sum_{t=1}^{T} \varepsilon_t^2}
$$

(6)

where $N$ is the number of observations.

The modeled price development according to Random Walk model is following

$$
P_t = P_{t-1} \times e^{\tau * dt}
$$

(7)

To estimate the price it is necessary to know the logarithmic price simulation, its mean value and dispersion. In case of Geometric Brownian motion with logarithmic prices, these calculations are relevant

$$
P_t = P_{t-1} \times e^{\tau * dt + \sigma * dz}
$$

(8)

$$
E(P_T) = P_0 \times e^{\tau * dt + \sigma \cdot \mu} = P_0 \times e^{\tau * T}
$$

(9)

$$
\sigma^2(P_T) = P_0^2 \times e^{2 \cdot \tau \cdot T} \times (e^{\sigma^2 \cdot T} - 1)
$$

(10)

The value of the quantile on the likelihood level $\alpha$ from log-normal distribution, which determines the limits within which should the random variables move, has the formula

$$
P_T^\alpha = P_0 \times e^{\tau * dt + \Phi^{-1} \cdot \alpha \cdot \sigma \sqrt{T}}
$$

(11)

Asset prices in financial markets behave randomly and independently of previous development, Brownian motion is thus an ideal tool to describe the behavior of asset prices.

3. EXCEPTIONS OF THE RANDOM WALK

Malkiel acknowledges some statistical anomalies pointing to some exceptions of the random walk theory:

- Prices of small, less liquid stocks seem to have some serial price correlation in the short-term because they do not incorporate information into their prices as quickly.
- Contrarian strategies$^2$ tend to outperform other strategies because reversals are often based on economic facts rather than investor psychology.
- There are seasonal trends in the stock market, especially at the beginning of the year and the end of the week.
- Stocks with low P/E ratios$^3$ tend to outperform those with high P/Es, although the tendency is volatile over time.
- High-dividend stocks tend to provide higher returns over time because during down markets$^4$ the high dividend yields often create demand for these stocks and thus increases the price.

$^2$ do not follow the general market trends,

$^3$ P/E ratio is an equity valuation measure defined as market price per share divided by annual earnings per share,

$^4$ down markets are designed for low-income consumers.
4. AN ELEMENTARY EXAMPLE OF RANDOM WALK

An elementary example of a random walk is the random walk on the integer number line. The starting point is at 0 and at each step moves +1 or −1 with equal probability (50 %).

This walk is explained as follows. A marker is placed at zero on the number line and a coin is flipped.

There are two possibilities of result. If the coin lands on head (H), the marker moves one unit to the right.

If the coin lands on tail (T), the marker changes its position one unit to the left.

After five flips, the market could land on the position of 1, -1, 3, -3, 5, or -5. With five flips, two tails and three heads, does not depend on the order, will the marker land on 1. There are:

- 10 ways of landing on 1 – by flipping three heads and two tails,
- 10 ways of landing on -1 – by flipping three tails and two heads,
- 5 ways of landing on 3 – by flipping four heads and one tail,
- 5 ways of landing on -3 – by flipping four tails and one head,
- 1 way of landing on 5 – by flipping five heads,
- 1 way of landing on -5 – by flipping five tails.

The following figure displays all possible random walk outcomes after five flips of a coin, where H – head, T – tail. In this case, the first landing is on head.
The next figure shows all possible random walk outcomes after five flips of a coin in case if the first landing is on tail.

CONCLUSION

Malkiel and the Random walk theory provide very useful and important support to the intimidated individual investor, but Malkiel in particular encourages investors to understand the theories and investment methods that the random walk theory challenges. Malkiel therefore advocates a buy-and-hold investment strategy. This represents the most effective way to maximize returns. All issues about capital market approaches could be explained by Malkiel’s famous quote: “Investment advisory services, earnings predictions, and complicated chart patterns are useless... Taken to its logical extreme, it means that a blindfolded monkey
throwing darts at a newspaper’s financial pages could select a portfolio that would do just as well as one carefully selected by the experts.”

BIBLIOGRAPHY


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Abstract

This paper describes and explains the fundamental characteristic of one of the theories that deal with stock prices estimation, called random walk theory. The term random walk was coined by Princeton economics professor Burton G. Malkiel in his book A Random Walk Down Wall Street in 1973. The random walk theory represents an investment theory which denies the influence of past price movements on the market prices and says that these prices follow a random path. This means that the market movements at any point are not possible to be predicted. In other words, this theory claims that stock’s prices path is random and cannot be determined on the ground of information gathered from historical prices, especially in the short-time horizon. This may be controversial, but by far the most controversial aspect of the theory is its claim that analysts and professional advisors add little or no value to portfolios.

The protagonists of random walk says that is not possible to outperform a financial market if no additional risk is taken on and explains why other analysis, such as technical, fundamental or psychological are not trustworthy or valid. These experts state that people often believe events are correlated if the events come in "clusters and streaks," even though streaks occur in random data such as coin tosses.

Authors:
Ing. Daniel Buc
Department of Economics
Faculty of Operation and Economics of Transport and Communications
University of Žilina in Žilina, 010 26 Žilina, Slovakia
E-mail: daniel.buc@fpedas.uniza.sk

doc. Ing. Tomáš Klieštik, PhD.
Department of Economics
Faculty of Operation and Economics of Transport and Communications
University of Žilina in Žilina, 010 26 Žilina, Slovakia
E-mail: tomas.kliestik@fpedas.uniza.sk